Scope Splitting and Cumulativity

Marcelo Ferreira

Universidade de São Paulo

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Complex Cardinal Quantifiers

(1) John kissed at least three girls.

(2) John kissed exactly three girls.

(3) John kissed fewer than three girls.

(4) At least four boys kissed at least three girls.

(5) At most four boys kissed more than eight girls.

Two Theories

(6) John kissed at least three girls.

✓GQ: $|\{x : girl'(x)\} \cap \{x : kissed'(\mathrm{j}, x)\}| \geq 3$

✓ PL: $\exists X : |X| \geq 3 \wedge girls'(X) \wedge kissed'(\mathrm{j}, X)$

GQ: Barwise and Cooper (1981)

PL: Link (1983) and many others after him

PL looks better

Cumulative Readings

(7) At least four boys kissed at least three girls.

$\sharp$ GQ: $|\{x : \mathit{boy}'(x)\} \cap \{x : x \text{ kissed at least 3 girls}\}| \geq 4$

OR

$|\{x : \mathit{girl}'(x)\} \cap \{x : \text{at least 4 boys kissed } x\}| \geq 3$

✓ PL: $\exists X \exists Y : |X| \geq 4 \wedge |Y| \geq 3$

$\wedge\ \mathit{boys}'(X) \wedge \mathit{girls}'(Y) \wedge \mathit{kissed}'(X, Y)$

GQ looks better

Non-Increasing DPs

(8) John kissed exactly three girls.

✓GQ: $|\{x : \mathit{girl}'(x)\} \cap \{x : \mathit{kissed}'(\mathrm{j}, x)\}| = 3$

♯ PL: $\exists X : |X| = 3 \wedge \mathit{girls}'(X) \wedge \mathit{kissed}'(\mathrm{j}, X)$

Neither looks good

Cumulativity with Non-Increasing DPs

(9) Exactly four boys kissed exactly three girls.

$\sharp$GQ: $|\{x : \mathit{boy}'(x)\} \cap \{x : x \text{ kissed exactly 3 girls}\}| = 4$

OR

$|\{x : \mathit{girl}'(x)\} \cap \{x : \text{exactly 4 boys kissed } x\}| = 3$

$\sharp$ PL: $\exists X \exists Y : |X| = 4 \wedge |Y| = 3$
$\wedge\ \mathit{boys}'(X) \wedge \mathit{girls}'(Y) \wedge \mathit{kissed}'(X, Y)$

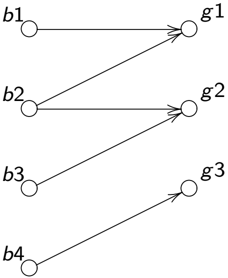
Fixing PL: Maximality

(10) John kissed exactly three girls.
$max\{n : \exists X : |X| = n \wedge girls'(X) \wedge kissed'(\mathrm{j}, X)\} = 3$

(11) $[\![$exactly three girls$]\!] =$
$\lambda P.\ max\{n : \exists X : |X| = n \wedge girls'(X) \wedge P(X)\} = 3$

Fixing PL?: Maximality and Cumulativity

(12) Exactly three boys kissed exactly two girls.
$max\{n : \exists X : |X| = n \wedge boys'(X) \wedge max\{n' : \exists Y : |Y| = n' \wedge girls'(Y) \wedge kissed'(X, Y)\} = 2\} = 3$



Interim Conclusion

(13) Exactly three boys kissed exactly two girls.
"The number of **boys who kissed girls** is 3 and the number of **girls kissed by boys** is 2."

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- When interpreting these sentences, we need access to the relation *boys who kissed girls*.
- The challenge is how to access this relation without violating compositionality.

Previous Proposals: Scha 1981

- There are binary determiners, such as *exactly3-exactly2* and binary nouns, such as *boys-girls* and binary quantification.
- Problem: there is no independent evidence for such unorthodox syntax.

Previous Proposals: Krifka 1999; Landman 2000

- Complex algorithms at the semantics-pragmatics interface that aggregate maximality claims to truth-conditions with the help of alternative semantics.

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- Complex algorithms at the semantics-pragmatics interface that aggregate maximality claims to truth-conditions with the help of alternative semantics.
- My point: There is no need for such radical moves, once we acknowledge that Complex Cardinal DPs have complex internal structures.

The Internal Structure of Cardinal DPs

- Hackl (2000): complex syntax for complex cardinal DPs

(14) $[_{DP}\ \emptyset\ [_{DegP}$ exactly 3 $[_{Deg'}$ Deg $[_{NP}$ boys]]]]
$[\![\text{boy}]\!] = \lambda x.\ boy'(x)$
$[\![\text{Deg}]\!] = \lambda P_{\langle et \rangle}.\lambda d.\lambda x.\ P(x)\ \&\ |x| = d$
$[\![\text{exactly 3}]\!] = \lambda P_{\langle dt \rangle}.\ \max\{d : P(d)\} = 3$
$[\![\emptyset]\!] = \lambda P_{\langle et \rangle}.\lambda Q_{\langle et \rangle}.\ \exists x : P(x)\ \&\ Q(x)$

Degree Operator takes Sentential Scope

(15) John kissed exactly 3 girls.
[$_S$ **exactly 3** [$_S$ λd [$_S$ John kissed [$_{DP}$ $\emptyset$ [$_{DegP}$ $\mathbf{t}_d$ [$_{Deg'}$ Deg [$_{NP}$ girls]]]]]]]
"The max. number *d* such that John kissed *d* girls is 3"

Degree Operator Movement and Cumulativity

(16) Exactly three boys kissed exactly two girls
$[_S$ ex. 3 [$\lambda d'$ [ex. 2 [λd [d' boys kissed d girls]]]]]

- Not Correct! "3 is the max. n such that n boys kissed exactly 2 girls"
- Incidentally, this is what Beck and Sauerland (2000:361) propose.

Degree Operator Movement and Cumulativity

(17) Exactly three boys kissed exactly two girls
$[_S$ ex. 3 $[_S$ ex. 2 $[_\alpha\ \lambda d\ [\ \lambda d'\ [\ d'$ boys kissed d girls]]]]]

- Syntactic note: we should allow the moved operators to stack on top of their lambda binders (see Sauerland (1998) and also Nissenbaum's (1998) analysis of parasitic gaps.)
- Semantic note: α denotes a degree relation, and it is possible to derive the cumulative interpretation based on the denotations of α, *exactly 3*, and *exactly 2*.

Degree Operator Movement and Cumulativity

(18) Exactly three boys kissed exactly two girls
$[_S$ ex. 3 $[_S$ ex. 2 $[_\alpha\ \lambda d\ [\ \lambda d'\ [\ d'$ boys kissed d girls]]]]]

(19) $[\![S]\!] = 1$ *iff*
$[\![\text{exactly 2}]\!](\lambda d.\exists d' : [\![\alpha]\!](d)(d'))$
&
$[\![\text{exactly 3}]\!](\lambda d'.\exists d : [\![\alpha]\!](d)(d'))$

The Cumulative Operator

- To complete the implementation, all we need is a "lifted" version of a cumulative operator

(20) $[\![\text{CML}]\!] = \lambda R.\lambda Q_1.\lambda Q_2.$
$$Q_1(\lambda x.\exists y : R(x)(y)) \;\&\; Q_2(\lambda y.\, \exists x : R(x)(y))$$

A more General Version

Non-Lexical Cumulativity: Beck and Sauerland (2000), but *cf.* Kratzer (2004)

(21) The (two) women wanted to marry the (two) men.

(22) Jim and Frank want to marry Sue and Amy (respectively)

A more General Version

(23) The (two) women wanted to marry the (two) men.

[the women [the men [$_\alpha$ λx [λy [x wants to marry y]]]]]

$[\![\text{the women}]\!] = \lambda P.\ P(\sigma(women))$
$[\![\text{the men}]\!] = \lambda P.\ P(\sigma(men))$

$$
\begin{aligned}
[\![\text{CML}]\!] = \lambda R.\lambda Q_1.\lambda Q_2.\ & Q_1(\lambda X.\forall x < X \exists y \exists Y \exists Z : y < Y \ \&\ Y \in Z\ \& \\
& \qquad Z \in min(Q_2)\ \&\ R(x)(y))\ \& \\
& Q_2(\lambda Y.\forall y < Y \exists x \exists X \exists Z : x < X\ \&\ X \in Z\ \& \\
& \qquad Z \in min(Q_1)\ \&\ R(x)(y))
\end{aligned}
$$

$$P \in min(Q) \leftrightarrow P \in Q\ \&\ P \neq \emptyset\ \&\ \neg\exists P' \in Q : P' \subset P$$

Scope Splitting

(24) You need to write at most five papers (to get promoted).

[$_S$ at most 5 [λd [you need to write d papers]]]

"There is no d greater than 5 such that you need to write d papers" (Hackl 2000)

- The sentence is about a minimal requirement.
- No specific papers should be written.

Scope Splitting and Cumulativity

(25) You need to donate at most twelve books to at most five public schools (to be eligible for tax deduction).

- The sentence is about a minimal requirement.
- No particular school should be given a particular number of books.
- No specific books or schools mentioned in the law.

Scope Splitting and Cumulativity

(26) You need to donate at most twelve books to at most five public schools (to be eligible for tax deduction).

[at most 5 [at most 12 [λd [$\lambda d'$ [you need [PRO to donate d books to d' schools]]]]]]

"There is no d greater than 12, such that you need to donate d books to public schools and there is no d' greater than 5, such that you need to donate books to d' public schools".

Conclusion

- A complex syntax for cardinal DPs provides the basis for a fully compositional analysis of cumulative readings with non-increasing DPs that does not require radical maneuvers either at the syntax-semantics or the semantics-pragmatics interfaces.

THANK YOU!

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